

A Channel Allocation Algorithm for OSA-Enabled IEEE 802.11 WLANs

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Abstract—Channel allocation problem is a major challenge in wireless local area networks (WLANs), especially in dense deployments of access points (APs) where congestion of the unlicensed spectrum bands (i.e., ISM bands) could undermine achieved network performance. This paper analyses the possibility to alleviate congestion of the ISM band by allowing some APs to use additional channels located in licensed bands in an opportunistic manner whenever licensee services (i.e., primary users) are not affected. Availability of these additional channels in licensed bands is assumed not to be the same for all the APs. Based on this assumption, we formulate the problem for the channel assignment as a Binary Linear Programming (BLP) problem, which allows us to obtain an optimal solution despite an elevated execution time. We also develop a heuristic method based on building a Minimum Spanning Tree (MST) graph attending to interference conditions that is able to find near-optimal solutions with a shorter execution time. Results are provided to assess the benefits of such a proposal under different WLAN deployment situations and primary channel availability conditions.

I. INTRODUCTION

Nowadays, the use of Wireless Local Area Networks (WLANs) based on the IEEE 802.11 standard is on the rise on both public (e.g., airports, train stations, leisure parks, etc.) and private premises (offices, hotels, home). As a matter of fact, about one third of Internet users in the U.S. make use of wireless local area networks, as reported in [1]. In this context, high dense WLAN deployments could lead to excessive levels of interference in the commonly used 2.4 GHz ISM band that could deteriorate network performance. High dense WLAN scenarios can arise from large-scale enterprise WLAN network deployments as well as a result of multiple individual WLAN installations in residential buildings (e.g., SOHO, Small Office Home Office use cases). Moreover, the potential adoption of WLAN-based mesh networks can also lead to a high concentration of Access Points (APs) in a limited geographical area. Hence, the availability of appropriate channel selection mechanisms constitutes an aspect of paramount importance to reduce the level of interference and allow a proper operation of such networks. Thus far, the channel allocation problem in WLAN has received a lot of attention in the research community [2], [3]. However, existing works have mainly focused on the development of

diverse channel assignment strategies that exclusively consider the channels available in the ISM band.

Unlike previous works, in this paper we analyze the possibility of alleviating the congestion of the ISM band in high dense WLAN deployment scenarios by allowing some APs to use additional frequency channels allocated to other services in an opportunistic manner. This spectrum usage concept is known in the literature as Opportunistic Spectrum Access (OSA) [4] and is supported by some recent spectral occupancy studies confirming very low spectrum occupancy of certain licensed bands [5]. This paper formulates the channel allocation problem in an OSA-enabled highly dense WLAN deployment as a binary integer programming (BLP) problem where interference levels between APs are kept below a certain interference threshold. It is considered that the APs can use a channel among those existing within the ISM band or, under some circumstances, a channel within an additional frequency band (i.e., primary band) licensed to other services (i.e., primary users). The conditions to determine the availability of these additional primary channels are considered per AP so that APs can have different primary band availability attending to the location and activity of the primary users (i.e., spectrum heterogeneity). Furthermore, it is considered that the usage of the primary band to alleviate congestion in the ISM band should be kept at the minimum extent possible, thus shielding as much as possible the channel allocation from the temporal and spatial variations of the primary channels' availability. The formulated channel allocation problem is solved by means of a heuristic algorithm based on the construction of a Minimum Spanning Tree (MST) graph attending to interference conditions. The proposed algorithm allows us to obtain near-optimal solutions with a reduced complexity. The performance of the heuristic algorithm is benchmarked against two other different approaches: a time-consuming branch-and-bound algorithm capable to find an optimal solution of the BLP problem (if feasible) and a random allocation of the frequency channels per APs (modelling the case where the channel configuration in each AP is done in an independent and uncoordinated way). Results are provided to assess the benefits of such a proposal under different WLAN deployment situations and primary channel availability conditions.

This paper is organized as follows. Section II describes the system model characterization. Then, in section III, the channel allocation problem is formulated as a BLP problem and the proposed heuristic algorithm is described. Results are provided in section IV and, finally, concluding remarks and future works are mentioned in section V.

II. SYSTEM MODEL

A. Network scenario

The considered network scenario consists of a set of individual APs (with their associated WLAN client stations) deployed in a limited geographical area. The channel used by each AP can be any ISM channel or a channel located in a licensed (primary) band whenever the usage of this channel does not affect the operation of the licensee, referred to as the primary user (PU). In this regard, the primary system is considered to co-exist with the WLANs in the same geographical area as shown in Fig. 1. Considering OSA notation, we refer to the APs (and associated stations) as secondary users (SUs).

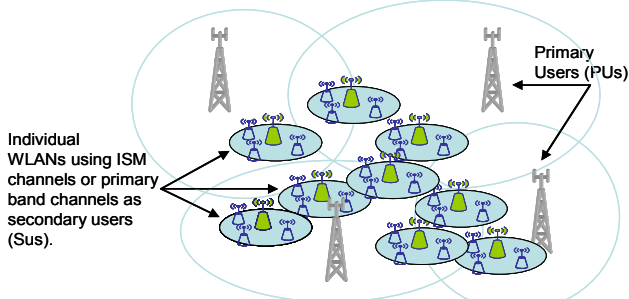


Fig 1. Network Scenario

B. Interference conditions

Conditions to determine which primary channels can be used by SUs are formulated in terms of the maximum interference levels that can be tolerated by both the PU and SU receivers. Hence, a SU can transmit in a certain primary channel whenever the interference received by any PU receiver tuned at that channel, I_{SP} , is below the PU receiver sensitivity S_P plus a given protection M_P . This usage condition imposed on SU transmitters can be formulated as $I_{SP} \leq S_P - M_P$. Additionally, the successful operation of SU receivers tuned into primary channels also mandates that the interference received from PU transmitters, I_{PS} , has to be lower than the SU receiver sensitivity plus a protection margin M_S . Hence, the usage condition required by SU receivers can be formulated as $I_{PS} \leq S_S - M_S$. It is worth noting that both receiver protection margins, M_S and M_P , would account for the fading margin along with the minimum required signal-to-interference ratio. Summing up, the possibility to use a given primary channel within a WLAN is subject to the accomplishment of both aforementioned usage conditions [6]. Notice that within a WLAN, the AP and its associated stations behave indistinctly as SU transmitters or receivers).

The two usage conditions can be used to define a set of usage and interference areas for PUs and SUs. Hence,

considering omnidirectional antennas, these areas would be circular shaped, as illustrated in Fig. 2. Assuming a single-slope propagation model with L_o being the channel attenuation at 1m and α the propagation slope, the radii of the usage area of the SU ($R_{U,S}$) and PU ($R_{U,P}$) can be calculated by means of the following equation:

$$R_{U,x} = 10^{\frac{(P_x - S_x) - L_o}{10\alpha}} \quad (1)$$

where the index x is either S or P so that the pair (P_x, S_x) stands for the transmitted power and receiver sensitivity of the correspondent system. Using the same propagation model, the computation of the radii of the interferences areas illustrated in Fig. 2 is quite straightforward from expressions given for the two usage conditions.

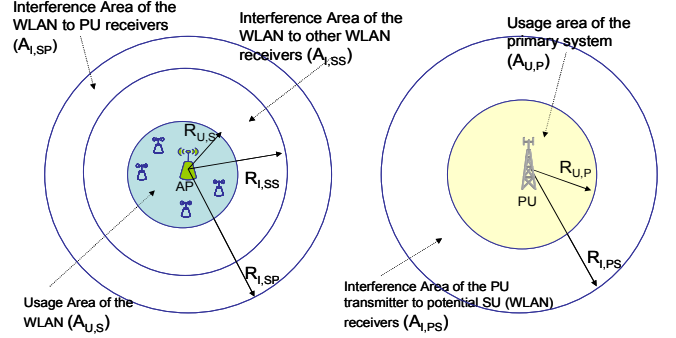


Fig 2. Usage and interference areas

The definition of these areas allows us to determine the possibility of using a given primary channel in an AP according to the accomplishment of the two following geometrical conditions: a) the usage area of the PU ($A_{U,P}$) using that channel is not overlapped by the interference area of the WLAN to PU receivers ($A_{I,SP}$), and b) the interference area of the PU transmitter using that channel ($A_{I,PS}$) does not overlap with the usage area of the WLAN ($A_{U,S}$). Notice that in the network scenario depicted in Fig.1 there will be dissimilar primary spectrum availability depending on the location of each AP.

C. Interference Penalty

The Interference Penalty (IP) is the metric used to quantify the interference level between a pair of individual WLANs and it is defined as follows:

$$IP_{m,n}(f_m, f_n) = \frac{A_{mn}}{A_{U,S}} \times \rho(f_m, f_n) \quad (2)$$

where f_m and f_n are the frequencies assigned to AP_m and AP_n respectively, A_{mn} is the overlapping area between the usage area ($A_{U,S}$) of AP_m and the interference area ($A_{I,SS}$) of AP_n , ($A_{U,S}$) is the usage area of AP_m and ρ is the overlapping channel factor defined in [2] for the IEEE 802.11 standard as:

$$\rho(f_m, f_n) = \max(1 - 0.2 \times |f_m - f_n|, 0) \quad (3)$$

III. CHANNEL ASSIGNMENT SCHEMES

In this section, we present two different channel assignment schemes aimed at keeping the interference level between each pair of APs under a given threshold. Both channel assignment

schemes consider that APs can always use any channel within the 2.4 ISM band in addition to other channels that may be available in an opportunistic manner in the primary band.

A. Optimal solution. BLP (Binary Linear Programming)

To achieve an optimal solution, we formulate the channel assignment problem as a Binary Linear Programming (BLP) problem [2] where the objective is to find the channel allocation for a given set of APs so that the interference penalty IP_{ij} measurement between any pair of APs (AP_i, AP_j) is below a certain threshold IP_{max} and the number of APs using channels in the primary band is minimized. The rationale behind pursuing the minimization of the primary band utilization is related to the need to find a solution with the lower dependability on the presence of primary users.

For every AP v , a channel f_i must be chosen among a set of potential channels F_{ISM} that contains all the ISM available channels plus an additional set $F_{PB}(v)$ with the primary channels specifically available for that AP. The order of available channels is considered so that the first channels, that is $i=1..|F_{ISM}|$, correspond to those in the ISM band and the subsequent ones, that is $i=|F_{ISM}|+1..|F_{PB}|$, are primary band channels. Hence, the channel selection in a given AP v is represented by means of binary variables defined as:

$$x_{v,f_i} = \begin{cases} 1 & ; \text{if } f_i, i=1..|F_{ISM}|+|F_{PB}(v)|, \text{ is assigned to } AP_v \\ 0 & ; \text{otherwise} \end{cases} \quad (4)$$

According to previous notation, the BLP for the channel allocation problem can be represented as follows:

$$\min \left(\sum_{v=1}^N \sum_{i=|F_{ISM}|+1}^{|F_{ISM}|+|F(v)|} x_{v,f_i} \right) \quad (5)$$

$$\text{s.t.:} \\ \sum_{i=1}^{|F_{ISM}|+|F(v)|} x_{v,f_i} = 1; \quad v=1,2,...,N \quad (6)$$

$$x_{v,f_i} + x_{w,g_j} \leq 1 \quad \text{if } IP_{v,w}(f_i, g_j) > IP_{max}, \text{ being} \\ i:1..|F_{ISM}|+|F(v)| \text{ and } j:1..|F_{ISM}|+|F(w)| \quad (7)$$

B. Near- Optimal Opportunistic Channel Allocation Solution

Next, we propose a heuristic algorithm used to assign a specific channel to each AP that meets the same objective fixed for the BLP problem. In this paper, this algorithm is called NOOCA (Near Optimal Opportunistic Channel Allocation). This algorithm is based on the minimum spanning tree (MST) problem, which is a well-studied problem in graph theory. Specifically, NOOCA is based on Prim's algorithm to find the MST of a given graph [7]. The WLAN scenario is represented as a graph $G=(V, E)$, where the nodes (vertices) $V=\{v_0, v_1, \dots, v_n\}$ are the APs and $E=\{e_0, e_1, \dots, e_m\}$ the edges between APs. Thus, the problem becomes: given a connected graph G and a weight $w: E \rightarrow \mathbb{R}^+$, we find an MST, and while doing so, assign to each node an appropriate channel. In particular, the weight on the edge is considered to be proportional to the overlapped area between the interference and usage areas of the two nodes (APs) connecting the edge.

The algorithm starts at the node that has the most overlapping area with respect to all the other nodes of the scenario. We define this node as x such that $V_{new}=\{x\}$. We set the channel of x to be the first channel from the ISM band ($f_x=1$) (line:1-2); From the neighbors of x , we choose the node that has the most overlapping area with x (u). The first time, there is only one channel assigned, and then u only has one secondary neighbor (SN). So, we give it a channel that is five channels apart from f_x . Hence, $f_x=6$ and $V_{new}=\{x,u\}$. We define the SNs as the neighbor nodes that have been assigned a secondary channel (Sch) (line:4); Then, the node u chooses the node with which it has the most overlapped area (v). So, if v has one SN (i.e. u), we give it a channel that is five channels apart from f_u (line: 6). If v has two SNs (u,w) (line:7-8), then we search for a channel that is five channels apart from (f_u, f_w) (line:9). If there are multiple solutions, we choose the one that causes the lowest IP . In the case of having three or more SNs we only consider the three with higher overlapping areas (u,w,y) (line:11). In this case we search for a channel that is " d " channels apart from the channels of the three SNs, d being the distance between the desired channel of the actual node and its neighbors (line:12); This d starts at five (i.e. without overlap of channels for 802.11 standard), but if it does not find any channel that satisfied this condition, we reduce by one unit until we find one (line:16-18). If there are multiple solutions we choose the one that causes the lowest IP ; Once we explore the possibility of using the secondary band for the current node, we calculate the IP caused between the current node and its SNs denoted as $IP_S(f_i, f_j)$ (line:20). If all the IP_S are below the fixed threshold (IP_{max}), we keep the channel chosen from the ISM band, or else we evaluate the possibility of using primary channels (Pchs) for the current node (line:21); If some of the $IP_S(f_i, f_j) > IP_{max}$ and the current node has available Pch (line:22), we search for the primary neighbors (PNs). PNs are the neighbor nodes to the current node that have been assigned a Pch assigned; If the current node does not have neighbors, the channel chosen will be any available Pch that it has. In the case that it has one, two or three PNs (k,z,y); we search for the Pch that is " d_p " channels from its PNs (i.e. d_p functions similarly to d). In the case of using the primary band, as this is not always available, there is the possibility that the only solution is to choose the same channel as one of its neighbors. In this case and in other possible cases, it will choose the channel that causes the lowest IP with its neighbors (line:23); Just as we did with the secondary band, we calculate the IP caused between the current node and its PNs denoted as $IP_P(f_i, f_j)$ (line:24) ; If all the $IP_P < IP_{max}$ (line:25) then the assigned channel will be one chosen from the primary band (line:33), or else we compare the sum of interference penalties between the current node and both PNs and SNs in order to assign the channel that produces the lowest interference penalty (line:27-31). This process is repeated for every node in order to assign a channel to each one (line:39,40). In Fig. 3, NOOCA algorithm is shown in detail.

Data: A connected weighted graph with vertices V and edges E .
Result: Minimal spanning tree composed on V_{new} and E_{new} where each $v \in V$ has an assigned channel.

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1.  $V_{new} = \{x\}$ , where  $x$  is the node that has the most overlapping area
   with other nodes from  $V$ ,  $E_{new} = \{\}$ 
2.  $f_v = 1$ 
3. while  $|V_{new}| \neq |V|$  do
4.   Choose edge  $(u,v)$  from  $E$  with maximum weight, such that
      $V_{new} = \{x,u\}$ 
5.   if  $v$  has only one neighbor then
6.      $f_v = f_u + 5 \bmod 11$ 
7.   else if  $v$  has only two neighbors then
8.     Let  $w$  be the second neighbor to  $v$  ( $u$  is the first neighbor)
9.     Choose  $f_v$  s.t.  $\min(|f_v - f_u|, |f_v - f_w|) = 5$ 
10.  Else
11.    Let  $w$  and  $y$  be the second and third neighbors with more
      weight to  $v$  ( $u$  is the first neighbor)
12.    Choose  $f_v$  s.t.  $\min(|f_v - f_u|, |f_v - f_w|, |f_v - f_y|) = d$ , where  $d=5$ 
13.    if  $f_v$  has multiple solutions then
14.      The channel that minimizes the  $IP$  is chosen
15.    end if (13)
16.    if  $d=5$  does not produce an assignment for  $f_v$  then
17.      Try  $d=4$ , then  $d=3$ , then  $d=2$ , then  $d=1$ 
18.    end if (16)
19.  end if (5)
20.  Calculate interference penalties of the neighbors with the
     secondary channel assigned to  $v$ 
21.  if  $(IP_S(f_v, f_u) > IP_{max}) \parallel (IP_S(f_v, f_w) > IP_{max}) \parallel (IP_S(f_v, f_y) > IP_{max})$ 
22.    if  $C_p \neq 0$  (it only assigns primary channel if it is available)
23.      Frequency assignment is calculated from primary
      channels in the same way that for secondary band (line 4-
      20). Also, if  $d_p=0$ , the channel assigned is the available
      primary channel which minimizes the  $IP$ .
24.      Calculate interference penalties of the primary neighbors
      with the primary channel assigned to  $v$ 
25.      if  $(IP_P(f_v, f_u) > IP_{max}) \parallel (IP_P(f_v, f_w) > IP_{max}) \parallel (IP_P(f_v, f_y) > IP_{max})$ 
26.        Let  $IP_{TOTP}$  be the sum of the secondary interference
        penalties and  $IP_{TOTP}$  (be the sum of the primary
        interference penalties)
27.        if  $IP_{TOTP} > IP_{TOTP}$ 
28.          The solution from the primary band is assigned
29.        else
30.          The solution from the secondary band is assigned
31.        end if (27)
32.      else
33.        The solution from the primary band is assigned
34.      end if (25)
35.    else
36.      The solution from the secondary band is assigned
37.    end if (22)
38.  end if (21)
39.  Add  $v$  to  $V_{new}$ , add  $(u,v)$  to  $E_{new}$ 
40. end while (3)

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Fig. 3. Opportunistic channel assignment algorithm.

IV. SIMULATION RESULTS

In this section, the proposed NOOCA algorithm is evaluated and its performance compared to the optimal solution derived from the BLP formulation and to another solution, referred to as random (RDM) that accounts for the usual way of allocating channels nowadays in real scenarios of independent WLANs: each WLAN selects its operational channel with no coordination with the rest. To solve the BLP problem, we use BINPROG from an optimization toolbox provided by MATLAB.

The evaluation is done by estimating the capacity of each allocation strategy (NOOCA, BLP, RDM) to find a feasible

assignment under a given network topology. To that end, we generate a topology snapshot by randomly placing primary and secondary users on a 1×1 area. Primary users randomly select a channel to use in the primary band. We consider $P_{ch}=8$ primary channels with similar transmission and power mask restrictions such as the ISM band [8]. Attending to the channel selection done by primary users, the available channel for each secondary user is obtained. Hence, depending on the location and spectrum used by primary users, APs can have available, in addition to $S_{ch}=11$ channels in the ISM band, between 0 and 8 channels in the primary band. Over such a basis, a channel assignment solution is calculated for each strategy. This process is repeated in a large number of snapshots, so that the percentage of feasible assignments (FA) achieved by each strategy can be assessed. Notice that a feasible assignment means that the resulting channel assignment is able to guarantee that the IP between each pair of APs is below the maximum allowed interference penalty (IP_{max}). Provided results have been obtained for 1000 snapshots. Radio operation parameters of the two systems have been chosen to have the following usage and interference area radii: $R_{U,S}=0.05$, $R_{U,P}=0.15$, $R_{I,PS}=0.3$, $R_{I,SP}=0.18$ and $R_{I,SS}=0.14$.

Fig. 3 illustrates the percentage of FAs achieved by BLP, NOOCA and RDM strategies attending to the target IP_{max} . As shown in the figure, when interference penalties are set to IP_{max} values above 0.5, both NOOCA and BLP achieve the maximum performance. On the contrary, the RDM strategy leads to non feasible assignments as soon as interference penalties are below 0.9. When stressing the interference guarantees of the allocation, i.e., low values of IP_{max} , some differences begin to appear between the proposed NOOCA and the optimal BLP solution. In the worst case considered, $IP_{max}=0.2$, NOOCA is able to find a feasible solution for 68% of the considered topologies, while the BLP is still able to cope with a feasible solution for each snapshot. However, the computation time required to solve the BLP is much higher than the one wasted by NOOCA. In particular, for $IP_{max}=0.2$, the computation time of BLP is above 0.5 hours while NOOCA algorithm only takes 30 ms on the same machine (a PC with processor Pentium IV of 3GHz and 1GB of RAM). Furthermore, results provided in Fig. 3 have been obtained under a limited scenario with a reduced number of users and channels (i.e., $SU=8$, $PU=0$, $S_{ch}=6$ and $P_{ch}=4$) in order to be able to provide results for the BLP solution.

Fig. 5 provides the percentage of FA values when considering different densities of APs ($SU=5..30$) and different numbers of primary channels ($P_{ch}=0,1,4,8$). Only NOOCA and RDM solutions are now compared due to the limitations to obtain solutions of the BLP problem in a reasonable time. As shown in the figure, when only considering ISM channels (i.e., no additional channels are assumed $P_{ch}=0$), NOOCA already achieves a considerably higher number of FAs than RDM (e.g., more than 70% for 10 APs). Besides, when additional channels are considered, the differences in NOOCA are even higher because the NOOCA is able to exploit these new channels more efficiently. Notice,

however, that the *FA* increase when considering primary channels is quite important when passing from $Pch=0$ to $Pch=1$, and from here, the increase is relatively lower. This is due to the fact that primary channels are also partially overlapped in frequency.

Finally, we consider the presence of primary users within the simulation scenario. These primary users make that not all the primary channels are now available for being used in all APs (i.e. spectrum heterogeneity). In Fig.6, results are provided for $Pch=4$ and the different numbers of PUs that can be using these channels, thus reducing their availability as potential secondary channels for APs. The main conclusion arisen from Fig 6 is that the number of PUs has less impact than expected in the benefits provided by NOOCA. The reason is that the availability of a single primary channel even in a reduced number of APs still provides enough margin to the NOOCA algorithm to find a good solution (i.e., the possibility of moving a single APs to the primary band can solve many interference problems).

V. CONCLUSIONS.

This paper has proposed a near-optimal heuristic algorithm for channel allocation in OSA-enabled WLAN networks. The channel allocation problem has been formulated as a BLP problem and the heuristic algorithm has been proved to obtain a significant number of feasible assignments with highly reduced computation complexity when compared to time-consuming branch-and-bound algorithms. The proposed NOOCA algorithm has been shown to efficiently exploit the heterogeneous availability of primary channels in a dense WLAN scenario so that mutual interference between individual WLAN can be reduced. Future work is ongoing to develop a distributed implementation of such type of algorithms between independent OSA-enabled WLANs.

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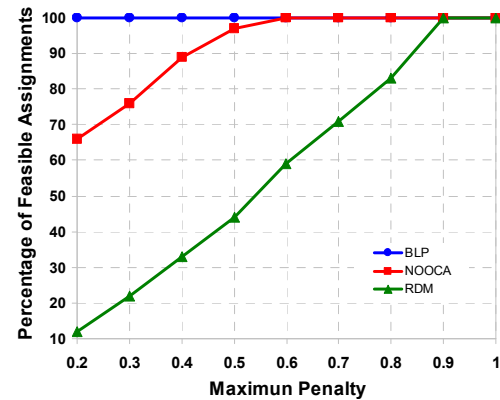


Fig. 4. Percentage of Feasible Assignments for BLP, NOOCA and RDM channel allocations versus IP_{max} for $SU=8$, $PU=0$, $Sch=6$ and $Pch=4$.

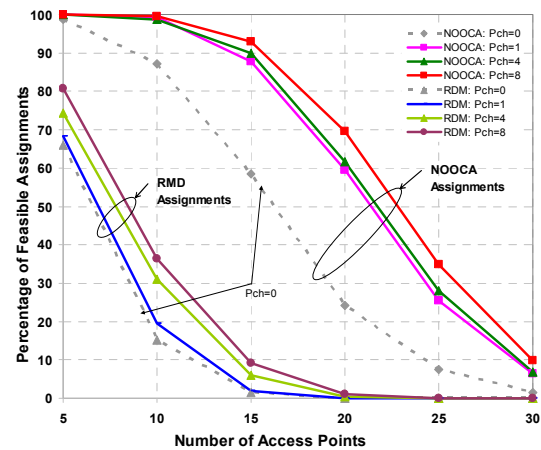


Fig. 5. Percentage of Feasible Assignment versus Number of Access Points for different number of primary channels. $PU=0$.

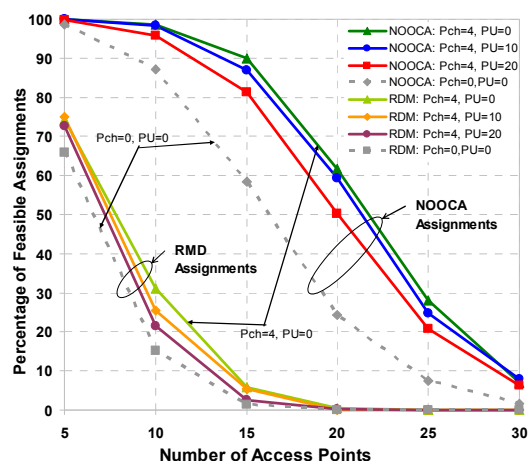


Fig. 6. Percentage of Feasible Assignments versus Number of Access Points for different number of primary users. $Pch=4$.